Flow-pressure analysis of loop gas networks

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***Abstract*—**This paper proposes a mathematical model underlying a computer program for flow-pressure analysis of loop gas pipe networks. The method is used on a test case with four nodes. The HAPN application for flow-pressure analyses of low pressure gas pipe networks is completely designed in object-oriented programming technology. The equations, which describe the physical flow-pressure conditions through every cross point are assumed to be continuous and the energy of every closed loop of analyzed network conserved. The system of non-linear equations was linearized by LTM (Linear Theory Method). The algorithm for numerical module LTM and the method for solution of sparse matrix are developed at the Faculty of Chemistry and Chemical Engineering, University of Maribor, Slovenia.

***Keywords*—**fluid mechanics, gas pipe networks, linear method, non-linear programming, pressure losses.

I. Introduction

THE energy criteria of production processes are nowadays becoming more and more important as a proper functioning of processes and their optimal exploitation ensure optimal functioning of devices together with a rational use of energy.

If we limit ourselves to the transport and distribution of energy in the gas phase in a supply network consisting of pipelines, linking customers with suppliers, we can safely say that the design and flow-pressure pipe network analysis is an inherent component in the complete design of transmission systems linked to optimal network installation that provides the greatest economic impact [1]. A significant characteristic of these systems is a large number of customers, large proportions of the various dimensions of pipes and the possibility of a large number of sources in the network.

Whereas in the short term with regard to the requirements of rational energy use and recommendations on resource use to ensure minimal environmental pollution an intense gasification is expected, it is essential to support the professional approach to planning, dimensioning and control of operating parameters of gas networks, which indisputably provides a great national benefit, not only from the urban, social and environmental aspects, but also from an economic perspective.

In the study we derive from the assumption that the configuration of the loop gas system is known; the aim of the study is to construct a mathematical model or computer program that allows pressure flow analysis and a professional approach to the realization of the introduction of gas as an energy product for broad consumption.

II. Natural Gas as an Energy Product

Natural gas is an energy product that has for its techno-economic characteristics the character of the most attractive energy source for industrial needs and for a broad and urban consumption. For a broad consumption in households, the consumption of natural gas is intended for heating, cooking and hot water. It is anticipated that an average household has the following gas-consuming devices:

- gas cooker with consumption of 1.25 m^3/h;
- gas kettle for preparing hot water with consumption of 1.25 m^3/h;
- gas boiler for the production of consumable hot water and for floor heating with the consumption of 3 m^3/h;

Since consumers of gas are usually remote, gas has to be supplied to consumers by pipelines. The most important quantities to be considered when sizing the pipeline are its:

- maximum operating pressure; and
- flow volume.

With respect to their operating pressure gas networks can be divided into:

- high-pressure gas networks (working pressure is higher than 1 bar);
- medium-pressure gas networks (working pressure from 0.1 to 1 bar);
- low-pressure gas networks (working pressure up to 0.1 bar).

When planning a gas network, the following must be carried out and observed:

- priority pipe network analysis;
- planning of optimal construction rules based on envisaged gas demand.

To determine the most important physical quantities and flow-pressure characteristics of the gas used in the network we use computer software. For the purposes of flow-pressure analysis, we specify:

- types of gas consumers;
- pressure load;
- the capacity of the network;

- friction coefficient;
- nominal pipe diameters;
- pipe connections;
- allowed speed, etc.

III. 3 The Prognosis of Natural Gas Consumption

The probability approach to long-term prognosis given as a result of predicted consumption with varying degrees of probability is broader, the more uneven its consumption had been in recent years [2].

Deviations of individual annual gas consumption from the regression function in the past cannot be considered as a random variable in the strict sense because the individual deviations from it are not totally independent of each other (e.g. economic developments in the previous year affect the consumption of gas in the next year). By prognosis we ignore selected random values and use individual consciously chosen values- by those it is meant the periodic sampling of the cumulative annual consumption. In doing so, the predicted trend is not necessarily accurate. The following assumptions are usually acceptable:

- due to the limited number of years taken into account we take Student's t-distribution of density probability;
- diversification of empirical time series from the trend, which is expressed by the variance, tends towards 0 for badly correlated annual consumptions if we increase the period of observation.

The following equation is valid:

$$\mathrm{var}(\overline{\varepsilon})\frac{\sigma^2\cdot\Delta t}{\tau_{op}}+\frac{2}{\tau_{op}}\cdot\sum_{k=1}^{n-1}(1-\frac{t_k}{T_{op}})\cdot K_\varepsilon(t)\cdot\Delta t \tag{1}$$

where:

$\overline{\varepsilon}$ - average deviations of the time series from the trend
σ - standard deviation
τ_{op} - observation period in years
Δt - T_{op}/years
t_k - $k^*\Delta T$
n - number of points in the sampling period during T_{op}
$K_\varepsilon(t)$ - covariance matrix - the values of the double sum

- probability interval along the regression line is constant in the observed past

If we limit ourselves to a linear regression, the mean square error is subject to the following equation:

$$\Delta^2 = E\cdot(Y-a\cdot t-b)^2 \tag{2}$$

where:

Y – annual gas consumption
a, b – parameter estimators
E – mathematical probability
t – time in years

Experience show that it is better to use empirically estimated values of the envisaged consumption in the target year, subject to the following equation:

$$y=f(t)=a+b\cdot t\pm \mathrm{var}(\overline{\varepsilon})\cdot r^{\frac{1}{z}} \tag{3}$$

where:

y – envisaged gas consumption
a, b – regression line parameters
z – forecast period

IV. 4 Flow-Pressure Losses

A. Basic Laws

The study considers the flow of natural gas in pipelines. The following assumptions are taken into consideration [3]:

- The flow is one-dimensional

In this case, we observe the gas flow in pipelines, where the relative change in intersection in relation to the length of flow in pipelines is low, where:

$$\frac{\partial D}{\partial x}\ll 1 \tag{4}$$

D is a characteristic dimension of flow. The change in flow parameters across the intersection along the flow is negligible. For such flows is the cross intersectional area a function of a single coordinate and that is the coordinates in the direction of flow $A = A(x)$.

- The flow is isotherm

Isothermal gas flow is created in the long un-insulated pipelines, where the amount of heat that the gas takes from its surroundings at the time of flow is sufficient to cover the temperature decrease due to expansion (Joule-Thompson effect).

- The flow is stationary

The temporal change of velocity and thermodynamic properties is negligible.

- Gas is a compressible fluid

The legalities of real gas movement are not fully consistent with the laws applicable to ideal gases. The deviation is given by the correction factor Z, called the compressibility factor, and is a function of pressure and temperature. It is given by the Berthelot equation:

$$Z=1+\frac{9}{128}\cdot\frac{p_R}{T_R}\cdot\left(1-\frac{6}{T_R^2}\right) \tag{5}$$

Where p_R is the reduced pressure:

$$p_R = \frac{p}{p_K} \tag{6}$$

And T_R reduced temperature:

$$T_R = \frac{T}{T_K} \tag{7}$$

P_K, T_K are critical state values.

- Friction coefficient is a function of wall roughness and Reynolds number.

The pipes used in flow systems are more or less rough. Roughness of the pipe varies in length and time. Absolute roughness of pipe (k) is the mean size of the roughness height on the inner surface of the pipe and depends on the type of pipe, method of manufacture, materials and the condition of the inner surface of the pipe.

B. Pressure losses in flat and inclined pipelines

When considering the conditions in the low pressure pipeline, we assume that the density during the gas flow does not change. The drop of pressure can be determined using the equation [4], [5]:

$$\mathrm{d}p = -\lambda \cdot \frac{\rho_1 \cdot v_1^2}{2 \cdot d} \cdot \mathrm{d}x \tag{8}$$

By integrating the equation (8) we get:

$$p_1 - p_2 = \frac{\lambda}{2} \cdot \frac{L}{d} \cdot \rho_1 \cdot v_1^2 \tag{9}$$

Equation (9) shows that in low-pressure pipelines with constant flow and constant gas density, the gas velocity is constant. In pipes that are not located horizontally, due to the difference in pressure between the air and transported gas buoyancy occurs. Pressure drop depends on the coefficient of linear losses, on the difference in density between the air and gas, and on the geodetic height Δh, subject to the following equation:

$$-\mathrm{d}p = \lambda \cdot \frac{\rho_1 \cdot v_1^2}{2 \cdot d} \cdot \mathrm{d}x + (p_z - p_1) \cdot g \cdot \frac{\Delta h}{L} \cdot \mathrm{d}x \tag{10}$$

where:

ρ_1 - is the gas density in point 1 (kg/m^3)

ρ_z - is the density of air (kg/m^3)

Δh - geodetic height between cross-sections (m)

$\Delta h > 0$ meaning the rise of the pipeline,

$\Delta h < 0$ meaning the fall of the pipeline.

By integrating the equation we get:

$$p_1 - p_2 = \lambda \cdot \frac{L}{2 \cdot d} \cdot \rho_1 \cdot v_1^2 + (\rho_z - \rho_1) \cdot g \cdot \Delta h \tag{11}$$

Considering the real gas state during streaming, depending on the length, pressure and density are reduced, whereas the speed is increasing. The flow of real gas in pipes is subject to the following equation:

$$R \cdot T = \frac{p_1}{z_1 \cdot \rho_1} = \frac{p_2}{z_2 \cdot \rho_2} = const. \tag{12}$$

where:

z_1 and z_2 are compressibility factors of gas in the initial and final state.

The compressibility factor (Z) depends only on the pressure during the isothermal change. Its mean value is calculated by the equation:

$$Z_m = \frac{1}{2}(Z_1 + Z_2) \tag{13}$$

Determination of pressure drop of real gas in high-pressure pipeline is subject to the following equation:

$$\frac{1}{p_1} \cdot \int_{p_1}^{p_2} p \cdot \mathrm{d}p = -\lambda \cdot \frac{\rho_1 \cdot v_1^2}{2 \cdot d} \cdot \frac{Z_m}{Z_1} \cdot \int_0^L \mathrm{d}x \tag{14}$$

or

$$\frac{p_1^2 - p_2^2}{2 \cdot p_1} = \frac{\lambda}{2} \cdot \frac{L}{d} \cdot \rho_1 \cdot v_1^2 \cdot \frac{Z_m}{Z_1} \tag{15}$$

1) Resistance factor of straight pipes

We introduce pipe resistance factor R for a quicker and easier determination of pressure losses. The value of R takes into account the normalized values of certain quantities of gas flow through the pipeline.

The loss of pressure in the high-pressure pipeline (p_1^2-p_2^2) bearing in mind the pipe resistance factor is:

$$p_1^2 - p_2^2 = R \cdot L \cdot q_v^2 \tag{16}$$

where:

L is the length of pipe (m), q_v is the volume flow by taking into account the normalized gas velocity in high-pressure gas pipeline (m^3/s).

$$R = \lambda \cdot \frac{Z_m}{Z_1} \cdot \rho_n \cdot p_n \cdot \frac{T_1}{T_n} \cdot \frac{16}{\pi^2 \cdot d^5} \tag{17}$$

In the low-pressure pipeline the deviation from the ideal state is not considered and the equation is as follows:

$$p_1 - p_2 = R \cdot L \cdot q_v^2 \tag{18}$$

where:

$$R = \lambda \cdot \rho_n \cdot \frac{p_n}{p_1} \cdot \frac{T_1}{T_n} \cdot \frac{8}{\pi^2 \cdot d^5} \tag{19}$$

2) Loop Resistance Factor

The analysis of parallel pipe connection and alternative pipe connection is in Fig. 1.

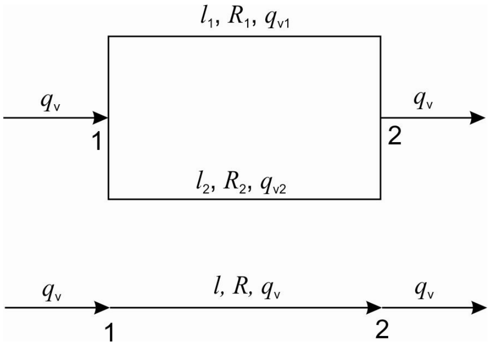

Fig. 1: Simple pipe network with two nodes and two pipes.

Between nodes 1 and 2 the condition

$$(p_1 - p_2) = \text{const.} \tag{20}$$

must be fulfilled, which means that the pressure drop between nodes 1 and 2 is identical in both sections of the output system and in the replacing pipe connection.

Assuming that

$l = l_1 \neq l_2$ and ($q_v = q_{v1} + q_{v2}$) we get:

$$R = \frac{R_1 \cdot R_2}{\left(\sqrt{R_1} + \sqrt{R_1}\right)^2} \tag{21}$$

The equation (21) allows the parallel piping system, which represents the loop, to be considered as a system of interconnected sections in the linear performance.

V. Mathematical Model

The term gas network means a larger number of individual interconnected pipelines, whose function is to supply gas to all consumers.

Network topology can be efficiently described and analyzed with graph theory, where gas network is being treated as a planar oriented graph. For further analysis the reading array of connection of pipes and nodes is essential. With incidence matrix a link to individual tubes with individual nodes can be written, which is interesting only when finding the optimal topology. To calculate the pressure in the individual network nodes we need e.g. Jacobian matrix of differential pressure flow, which also takes into account the topology and the physical properties of fluid in the pipeline. Before that, the flow directions need to be assumed.

The custom configuration of the gas network consists of L tubes and N nodes. Nodes represent the places in which the individual pipelines merge or diversify, or places in which the fluid enters or leaves the network. The branches represent the connections between nodes (Figure 2). They are described by the length, inner diameter, and volumetric flow, the direction of flow and pressure losses [6].

Loops represent the branching and merging of the pipeline. The number of elementary loops depends on the number of branches and on the number of nodes, subject to the following equation:

$$m = s - v + 1 \tag{22}$$

where:

m- number of loops
s – number of branches
v – number of nodes

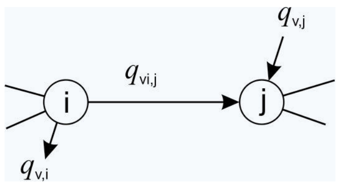

Fig. 2: Diagram of branch with vertices i and j

The flow is considered as positive if it is entering the node, and as negative if it flows from the node. For each node it is valid that the sum of the flows is equal, meaning that the sum of the flows at the entrance to the node equals the sum of flows at the exit of nodes. 1st Kirchhoff law applies, which is described by the linear algebraic equation [7]:

$$\begin{aligned} & q_{v1} - \sum_{i=2}^{N_v} q_{vi} = 0 \\ & \sum_{j=1}^{N_c} \pm q_{vij} \pm q_{vi} = 0 \qquad (i = 1,2,3,\ldots,N_v) \\ & q_{vij} \geq 0 \qquad (i = 1,2,3,\ldots,N_v \; j = 1,2,3,\ldots,N_c) \end{aligned} \tag{23}$$

Also, the sum of pressure losses in any closed piping network loop equals zero. 2nd Kirchhoff law applies, which is described by the nonlinear algebraic equation:

$$\sum_{I=1}^{N} \Delta p_i = 0 \tag{24}$$

In the case of a large number of pipelines and loops, for solving a system of nonlinear algebraic equations the following numerical methods are used [8]:

- iteration method;
- Newton's method;
- the method of functional iteration;
- Cauchy method;
- Hook-Jeeves method;
- the method of quadratic programming;
- modified Newton's method;
- linear method;

The study gives a mathematical model whose solution is based on the use of linear methods and computer.

The solution gives remarkable results, which are characteristic of fast convergence. One can also easily respond to the questions of how to maintain the optimal gas pressure in spite of variable consumption; what are the consequences caused by additional power supply of the network, or the change of direction of gas flow in pipes in case of compliance with the legality of stationary incompressible flows.

A. Linear Method [9], [10]

The loop- tree network (Fig. 3), node 1, is subject to the continuity condition:

$$-q_{v1,2} + q_{v1,3} - q_{vv1} = 0 \tag{25}$$

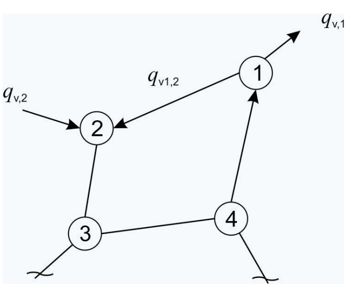

Fig. 3: Loop-tree network

The values of volume flows are subject to the equation:

$$-\sqrt{\frac{p_1 - p_2}{r_{1,2}}} + \sqrt{\frac{p_3 - p_1}{r_{1,3}}} - q_{v1} = 0 \tag{26}$$

If we multiply equation (25) in numerator and denominator with $\sqrt{p_i - p_j}$ we get:

$$-\frac{p_1 - p_2}{\sqrt{r_{1,2} \cdot (p_1 - p_2)}} + \frac{p_3 - p_1}{\sqrt{r_{1,3} \cdot (p_3 - p_1)}} - q_{v1} = 0 \tag{27}$$

(26) can also be written as:

$$(p_2 - p_1) \cdot C_{1,2} + (p_3 - p_1) \cdot C_{1,3} - q_{v1} = 0 \tag{28}$$

$$-p_1 \cdot (C_{1,2} + C_{1,3}) + p_2 \cdot C_{1,2} + p_3 \cdot C_{1,3} - q_{v1} = 0 \tag{29}$$

where:

$$C_{i,j} = \frac{1}{\sqrt{r_{i,j} \cdot (p_i - p_j)}} \tag{30}$$

Continuity equations can be written for all nodes, and we get a system of linear equations. Coefficient Ci,j is calculated by assuming pi and pj. After first calculation of the linear equations system, we get approximate values for the pressures at nodes that are used in the next calculation of Ci,j. The calculation procedure is repeated until we reach the required relative accuracy.

VI. Computer Algorithm

With respect to above-mentioned theory was in terms of finding the most appropriate numerical method for hydraulic analysis of gas pipeline networks developed a computer algorithm using numerical:

- LTM linear method; and
- Newton - Raphson method [11].

Flowchart of computer algorithm to calculate the flow-pressure conditions of gas network by a LTH-method is shown in Fig. 4 [12].

The index k means the k-to iteration, X_{MAX} is the required relative accuracy. In the first iterative step, we assume the pressures in nodes $p_{\mathrm{i(k)},}$ then we change the value of Δp in each iterative cycle, which are obtained by solving the system of equations. The total number of iterations is dependent on the required relative accuracy [13].

A. Program Package

A software package is designed for user-friendly and with menus supported implementation of programs [14]. The package can simulate any gas supply system and its operating conditions, and can determine the operating conditions for the desired load. In comfortable dialogue the capture and alteration of data is enabled, as well as correction and implementation of accounts, and the display of results.

The program allows for a great flexibility; it is designed for engineers, planners and operators. To compile the data and to carry out programs no knowledge of computer science is required. The mathematical model does not require any restrictions on the size of the network system, and the number of loops or elements in the network. Each node can be within the capabilities of computer memory connected by any number of other nodes.

Consumers and energy sources are connected to the node, while the control and regulating devices are installed as elements into sections. They redirect and shape the fluid parameters. Consideration of items in a computer model is simple and possible in the context of programs for data entry [15].

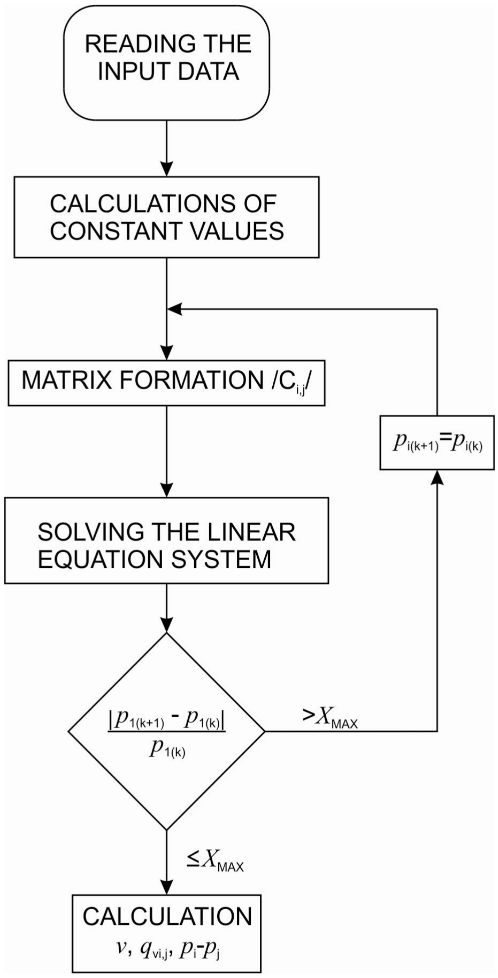

Fig. 4: Flowchart of computer algorithm

When entering the data, the user of the software package does not need to watch which pipes compose specific loops; also, it is not necessary to determine a positive direction for the first estimate of flows in these directions.

Apart from topological description of the network, also the technical characteristics of the elements to qualitative and quantitative define production, transport, distribution and consumption of energy are given.

Physical properties of the transported fluid are defined by the initial temperature and pressure, and are calculated for a particular situation based on tabular values.

Nodes, pipes and elements are entered in any order, whereas system data and parameters of temperature medium are prepared just before each calculation. Repeating the calculation is performed easily only by changing certain system information or by setting the proper position of valves in the network.

Input data are divided into data that give the network structure, the information on consumption of the customers, and the data on system parameters that define the current calculation. The bases for preparing and data entry are plans which provide a complete and current picture of network system with the connected and possibly planned customers.

The implementation of the applications is enabled by the following data groups:

- system data (physical properties of gas);
- nodes (consumption, geodetic altitude, coordinates x, y);
- tubes (length, diameter, roughness, embedded components or local resistors);
- sources (characteristics, capacity).

The screenshots for basic data entry, data on nodes and pipes, and the display of results is shown in the test case in Fig.5 and in Table 1.

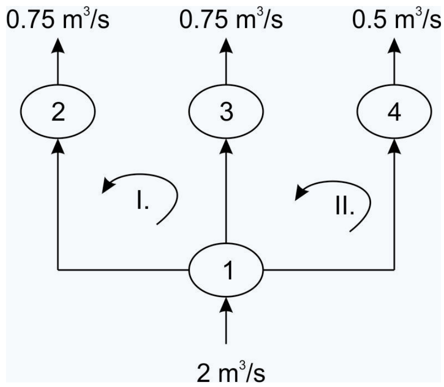

Fig. 5: The network of a test case.

Table 1: System data and results.

SYSTEM DATA	
Number of nodes	4
Number of pipes	5
Relative accuracy	1.00E-05
Max number of iteration	100
Kinematic viscosity (m^2/s)	1.42E-05
Gas density (kg/m^3)	8.40E-01

NODE DATA			
Node	Inlet-outlet (m^3/s)	Hydraulic level (m)	Node press. (Pa)
1	0.00	0.0	2600
2	-0.75	0.0	0.0
3	-0.75	0.0	0.0
4	-0.50	0.0	0.0

PIPE DATA						
from	to	Dz(mm)	Dn(mm)	L(m)	Ceta	Roughness
1	2	0.0	200.0	50.0	0.0	1.0
1	3	0.0	300.0	79.0	0.0	1.0
1	4	0.0	250.0	47.0	0.0	1.0
3	2	0.0	300.0	45.0	0.0	1.0
4	3	0.0	250.0	42.0	0.0	1.0

RESULTS					
from	to	q(m^3/s)	v(m/s)	hp(Pa)	dp(Pa)
1	2	0.40339	12.84	0.0	534
1	3	0.90038	12.73	0.0	492
1	4	0.69622	14.18	0.0	458
3	2	0.34661	4.904	0.0	43
4	3	0.19623	3.998	0.0	34

Node	Pressure(Pa)
1	2600
2	2066
3	2108
4	2142

I. Conclusion

Determination of flow-pressure conditions for gas supply networks is not straightforward and transparent and requires considerable computational effort to monitor the situation in each point of the network. The exact mathematical calculation without a computer is almost impossible. Software package includes a set of computer programs that are composed of independent modules and is designed to determine

the steady flow conditions in an arbitrary shaped loop networks and various pipelines.

The defined operating conditions with known loads and network architecture enable to specify the pressure, volume and speed conditions at every point of the network.

On this basis the following is possible:

- the designing and locating of energy production, transport and distribution devices, which are included in the system;
- the designing of new sections of piping systems;
- the defining and locating of critical points in the network, overloaded or under loaded sections;
- having full access to the existing flow-pressure conditions.

Using the software package is possible when:

- planning the supplying strategies;
- determining the optimal operation of the network;
- planning and designing.

Engineers and designers can use a software package for a fast and reliable identification and verification of the effects of operating conditions. The program can simulate any supply system and the operating conditions and calculates operating conditions for the current load. Based on the analysis of the results we can determine the most economical supply options for consumers and service network [16], [17].

The functionality of designing and dimensioning of new networks or network parts, the envisaged expansion of existing networks, thereby increasing consumption, as well as installation of new devices, are checked before the investment is triggered, by carrying out a flow-pressure analysis. The analysis gives directions for the construction and operation of the system. To this end, we need algorithms supplemented by boundary conditions or criteria. By known consumption and maximum allowable pressure drop it is possible to determine the appropriate diameter for all sections. On the basis of standard diameters and taking into account other planning criteria we can select the optimum pipe dimensions. Before calculating it is possible to define specific sections with pre-selected diameter, and they remain unchanged until the end of the calculation.

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