

Partially decoupled DMC system

P. Skupin, W. Kłopot, T. Kłopot

Abstract— This paper presents on the application of the partial decoupling control for the nonlinear MIMO hydraulic plant composed of two tanks connected with each other. In order to make the plant more challenging for control the first tank has variable cross sectional area and there has been applied additional interaction for input flow streams. By means of the three port valves some fraction of the first flow stream (F_1) is fed into the second tank and some fraction of the second flow stream (F_2) is fed into the first tank resulting in the smaller relative degrees in the cross channels of the nonlinear plant. The goals of the control algorithm were maintenance of the both liquid levels (H_1, H_2) at preset values and elimination of the interactions between control loops. The application of the partial decoupling matrix of constant elements together with the DMC controller for the first tank and PI controller for the second tank (of constant cross sectional area) gave satisfactory results – smaller interactions between control loops. Moreover, it has been studied the quality of control in the presence of the additional disturbances for the system with active or non-active decoupling mechanism. The plant was disturbed by two oscillating sinusoidally with nonzero mean input flow streams (F_3, F_4) for the first and second tank respectively. The simulation results have shown that the decoupling mechanism improves quality of control in the presence of the disturbances.

Keywords— Adaptive control, DMC (Dynamic Matrix Control), Partial decoupling, Predictive control, Nonlinear plant.

I. INTRODUCTION

MANY of the technological lines base on sophisticated tanks connections. These connections and precisely chosen shapes of tanks with additional technological guidance assure of achieving process goals. On the other hand, it seems that liquid level control should not be a challenge for control algorithms. However, some specific shapes of tanks frequently used in industry are in opposite to this statement. One of the shapes which imposes great requirements on process control is the conical tank which is widely used in industry i.e. food process industries, hydrometallurgical industries and especially in water treatment industries, where it improves disposal of solids while mixing. The paper presents plant (Fig.1) which is

Manuscript received July 15, 2010; Revised version received July 15, 2010. This work was supported by the Polish Ministry of Scientific Research and Higher Education.

P. Skupin is with the Faculty Of Automatic Control, Electronics And Computer Science, Silesian University of Technology, Gliwice, 44-100 Poland, (e-mail: piotr.skupin@polsl.pl).

W. Kłopot is with the Faculty Of Automatic Control, Electronics And Computer Science, Silesian University of Technology, Gliwice, 44-100 Poland, (e-mail: witold.kłopot@polsl.pl).

T. Kłopot is with the Faculty Of Automatic Control, Electronics And Computer Science, Silesian University of Technology, Gliwice, 44-100 Poland, (e-mail: tomasz.kłopot@polsl.pl).

difficult to control because of: the shape of the first tank (non-linear), interconnection of the tanks and feed system (stronger cross couplings). Taking under consideration these features presented paper focused on an increase of control quality by implementation of advanced control algorithm [1]. Therefore, adaptive DMC controller [2], [3] with decoupling mechanism has been applied. Additionally, in the article two cases were compared, where decoupling mechanism was active and inactive.

II. PLANT DESCRIPTION

The studied hydraulic tank system is composed of two coupled tanks - conical and cylindrical tank. The third tank in the system plays buffering role and it develops closed water flow loop. For that reason, the whole real pilot plant can be treated as an energy saving plant. The operating point is set by the three valves (V_1, V_2, V_3) - by changing their hydraulic resistance and the outflows are assumed to be turbulent.

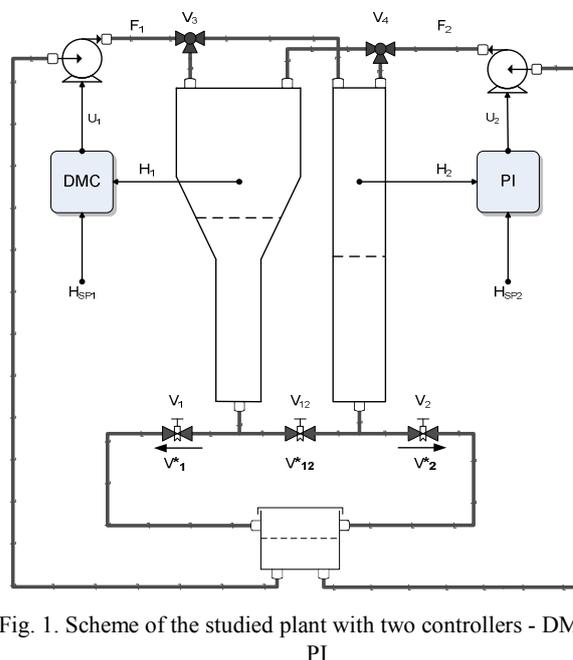


Fig. 1. Scheme of the studied plant with two controllers - DMC and PI

This assumption makes studied plant more difficult to control. In order to have more precise influence over plant cross-couplings and their strength it was decided to split up inflows by means of the three-port valves which are set prior to the simulation experiments and their parameters can be changed in the range $\zeta_i [0,1]$, ($i=1,2$). In such case the whole inflow to the conical tank will be described by sum of $(1 - \zeta_1)F_1$ and $\zeta_1 F_2$,

where F_1, F_2 - flow rates for the first and the second pump, respectively. During simulation investigations, both parameters ζ_1 and ζ_2 were assumed equal to 0.3. Such an assumption entails that the relative degrees in the main and coupling channels are equal to unity and then there appear much stronger interactions between control loops than in the case without splitting up the input flows, i.e. for $\zeta_1=\zeta_2=0$. Furthermore, if $\zeta_1=\zeta_2=0$ then the relative degrees in the main channels are equal to unity and in the cross channels are equal to two (definition of the relative degree for nonlinear plant – see [4]). Similar solution was applied for quadruple-tank process described in [5]. A schematic diagram of the process is shown in Fig.1. The aim is to control the liquid levels in the tanks with the help of two pumps. The process outputs are H_1 and H_2 (liquid levels), whereas the inputs are U_1 and U_2 (pump control signals). The DMC controller maintains the level H_1 at pre-set level H_{SP1} and the PI controller maintains level H_2 at pre-set level H_{SP2} . The whole system was simulated in LabView environment which provides real-time simulations and, moreover, the considered tanks are part of the real laboratory installation [6]. The investigated hydraulic tank models were validated using experimental data.

III. MATHEMATICAL MODEL

According to the law of conservation of mass, general equation can be written as follows:

$$\frac{dV}{dt} = F_{IN} - F_{OUT},$$

where:

$$\frac{dV}{dt} = \frac{dV}{dh} \frac{dh}{dt} = A(h) \frac{dh}{dt}$$

and:

$$A(h) \frac{dh}{dt} = F_{IN} - F_{OUT} \quad (1)$$

Regarding to equation (1) it can be written:

$$A_1(h_1) \frac{dh_1}{dt} = (1 - \zeta_1)F_1 + \zeta_2 F_2 - kv_1 \sqrt{h_1} - kv_{12} \sqrt{h_1 - h_2} \quad (2)$$

$$A_2(h_2) \frac{dh_2}{dt} = (1 - \zeta_2)F_1 + \zeta_1 F_2 - kv_2 \sqrt{h_2} + kv_{12} \sqrt{h_1 - h_2} \quad (3)$$

Where:

$$A(h_2) = \frac{A(H_2)}{A_{2,max}} = 1 \quad (4)$$

$$A(h_i) = \frac{A(H_i)}{A_{i,max}} = \begin{cases} \frac{1}{\left(\frac{D}{d}\right)^2}, & \text{for } H_i < h_{R1} \\ \frac{1}{\left(\frac{D}{d}\right)^2} \left(1 + \left(\frac{D}{d} - 1\right) \cdot \frac{H_i - h_{R1}}{h_{R2} - h_{R1}}\right)^2, & \text{for } h_{R1} < H_i < h_{R2} \\ 1, & \text{for } h_{R2} < H_i < h_{R3} \end{cases} \quad (5)$$

Where:

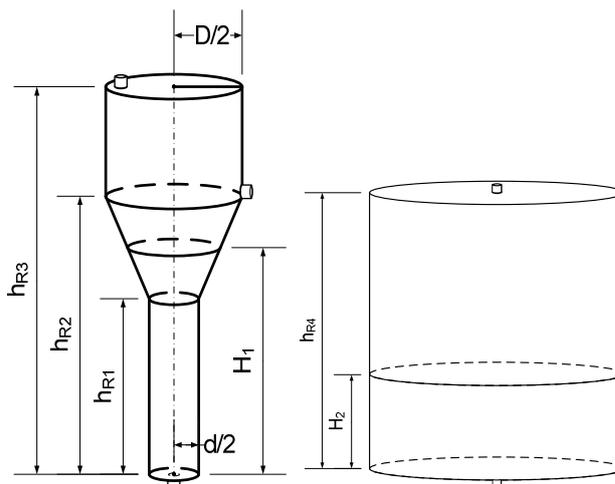


Table Table1. Used nomenclature.

Symbol	Description
F_1, F_2	output flow rates
$A_{i,max}$	maximal i-th tank surface area
$A(h_i)$	normalized surface area to $A_{i,max}$
$h_1 = H_1/h_{R3}$	normalized current liquid level in the conical tank
$h_2 = H_2/h_{R4}$	normalized current liquid level in the cylindrical tank
$h_{R1}, h_{R2}, h_{R3}, h_{R4}, D, d$	real plant geometrical dimensions

IV. CONTROL ALGORITHM

For higher efficiency of the control results the commonly known DMC algorithm was used [7], [8]. Taking under consideration non-linear plant characteristic the DMC controller was tuned in five sections. Fig.2 presents the way of partition. Tuning process applies step response in each section. Obtained data has been used to tune DMC controller according to the tuning sequence proposed by [9], [10].

Sequence is as follows:

- STEP1. Select the sample time T

$$T = 0.1T_1$$
- STEP2. Compute the prediction horizon HP

$$H_p = \text{round}\left(\frac{T_1}{T}\right) + \text{round}\left(\frac{T_0}{T}\right)$$
- STEP3. Compute the control horizon HC

$$H_c = 2$$
- STEP4. Compute the model horizon HD:

$$H_D = \text{round}\left(\frac{2T_1}{T}\right) + \text{round}\left(\frac{T_0}{T}\right)$$
- STEP5. Compute the move suppression coefficient λ :

$$\lambda = H_p \cdot K^2 \cdot x$$

where:

- x - fine tuning parameter,
- T_0 - process dead time,
- T_1 - process time constant,
- K - steady state process gain.

As it was mentioned earlier, for simulation purpose the conical tank has been divided into five parts where the conical part of the tank was divided into three sections. Dividing idea is widely known and used [11], [12]. Observations show there is no need to increase quantity of sections in the studied tank. It is obvious that the higher number of divided parts the higher computational cost. Consequently, the higher accuracy is obtained [11]. However, received gain is too low to be taken into consideration. Therefore, it was decided to use three sections. In each section, the DMC controller was tuned.

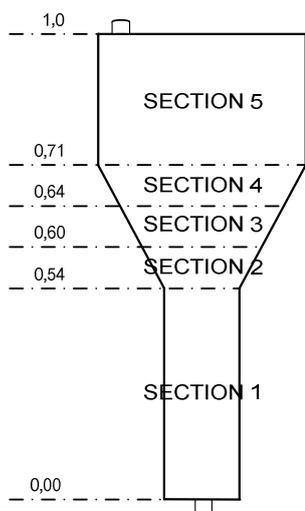


Fig. 2 The conical tank divided on sections.
 Scale normalized to h_{R3} .

Additionally the DMC controller has been tuned above and below the cone. Depending on the present liquid level, appropriate matrix coefficients are used - in the same way as in the gain scheduling for PID control [13].

V. DECOUPLING ALGORITHM

As mentioned earlier, in multivariable plants, there are cross-couplings that may make remarkably difficult the controller synthesis, moreover, such cross-coupling may also make remarkably difficult the quality of control (especially the quality of transients) due to harmful influence of the i -th control signal on the remaining j -th ($j \neq i$) output signals of the plant. With regard for the interactions, there have been developed series of methods leading to the full or partial decoupling of the plant. Most of these methods are developed, first of all, for linear systems and, in the literature, the most familiar methods are those connected with static decoupling (elimination of the interactions only in steady states) and with fully dynamic decoupling by means of dynamic element $D(s)$ connected in series [14], [15] or by means of state feedback for complete compensation of interactions between control loops [16], [17]. In turn, for nonlinear systems the non-interacting control is mostly achieved via state feedback ([18], [19], [20], [21]). However, in the case of the mentioned methods, it is required the exact model of the plant or the availability of state variable measurement what in real life may not be possible. Furthermore, in the case of the full dynamic decoupling by means of the dynamic element (matrix $D(s)$), its form may be quite complex, what to some degree, may also make difficult the practical realization of the method. There are also exist methods for which the decoupling matrix $D(s)$ do not contain any dynamical elements [22] concurrently ensuring almost complete cross-couplings compensation. The example of such a method is the method of partial decoupling of fast transients proposed in [23], which consist in the appropriate choice of the decoupling matrix D_f of constant elements which is connected in series between the controller and the plant. Assuming that the given plant has n -input signals and n -output signals and is described by the matrix transfer function $K(s)$ the unknown decoupling matrix D_f should ensure such form of the modified plant $K_z(s) = K(s)D_f$ that $K_z(s)$ has on its diagonal the elements with the smallest relative degree in each row. Thanks to such form of the decoupled matrix transfer function of the plant $K_z(s)$, i -th control signal has the stronger influence on the i -th output signal of the plant and smaller influence on the remaining output signals. Thorough investigations of the method have shown that the faster transients are the smaller interactions between control loops [23], [24]. In the present work it will be shown that the use of described partial decoupling method developed for linear systems also gives quite good results for the given nonlinear plant (2), (3). For the purpose of finding the decoupling matrix D_f for the nonlinear plant (2), (3) we assume the linear time invariant form of the plant given by the matrix transfer function $K_0(s)$ found at the

chosen operating point ($h_{10}=0.6, h_{20}=0.5$):

$$\mathbf{K}_0(s) = \begin{bmatrix} \frac{0.4533}{0.4s+1} & \frac{0.9065}{1.5s+1} \\ \frac{0.3534}{s+1} & \frac{0.7068}{0.9s+1} \end{bmatrix} \quad (6)$$

The dynamics of the main and coupling channels are described by elements of first order, where time constants are so chosen to ensure the best fitting to the step responses of the nonlinear plant around the assumed operating point. In turn, gain values are so chosen to ensure the identity between appropriate output signals of the plants at the assumed operating point. In that case all the component elements of the transfer matrix function $\mathbf{K}_0(s)$ are elements of first order with the same relative degrees equal to unity what correspond with the same relative degrees of the nonlinear plant (2), (3). For the linearized plant (5) there exists matrix \mathbf{D}_f of constant elements decoupling fast transients:

$$\mathbf{D}_f = \begin{bmatrix} 1.1610 & -0.8934 \\ -0.5225 & 1.6754 \end{bmatrix} \quad (7)$$

The detailed description of the calculation of the decoupling matrix \mathbf{D}_f can be found in [23]. For the chosen decoupling matrix (6) numerous investigations have been carried out to show differences between the closed loop systems with DMC and PI controllers with and without partial decoupling.

VI. SIMULATOR DESCRIPTION

For the simulation experiments, one-step Euler algorithm was applied. The differential equations (2), (3) were approximated using the forward difference method. Obviously, it is possible to use other integration algorithms such as: Runge-Kutta algorithm, as well as multi-step Adams-Bashforth, Adams-Moulton and Adams-Bashforth-Moulton algorithms, however, they are more complex. Simulation bases on the UDYN (Unify DYNAMics) idea [25], [26], where the integration step size h is defined by the NIS value. The NIS value represents the number of integration steps in the Dt interval.

$$h = \frac{Dt}{NIS} \quad (8)$$

The plant simulator and both controllers were programmed in LabView environment, which provide real time simulations. Consequently, the control system is free of all kind of noises and disturbances. Simulation also provides such additional properties as the repeatability of measurements and time invariance of the plant. Since the plant and controllers were simulated there can be done objective comparison between systems with and without partial decoupling algorithm. The

obtained results are presented in the next section. In simulation natural limitations such as: signal control range and maximal volume of tank were considered by cutting signal value to allowed range. More sophisticated methods are described in [8]. All outflows are assumed to be turbulent. This assumption in conjunction with the nonlinearities (mainly caused by shape of the conical tank) makes studied plant very challenging for control algorithms.

VII. RESULTS

For the system with nonlinear plant described by (2), (3), with the DMC controller for the conical tank and with the PI controller for the constant cross section tank, there have been carried out simulation investigations to compare the quality of control for closed loop systems with and without partial decoupling by changing both set points (H_{SP1}, H_{SP2}). The same tuning parameters of the DMC and PI controllers were used in both cases (with and without decoupling) in order to only show the influence of the partial decoupling algorithm on the quality of transients. For the better evaluation of the obtained results the following performance index is proposed which is calculated for each control error separately over some time interval $[t_1, t_2]$ comprising all the set point changes:

$$Q_i = \int_{t_1}^{t_2} e_i^2 dt \quad (9)$$

where e_i – denotes the control error for the i -th process variable ($i=1,2$).

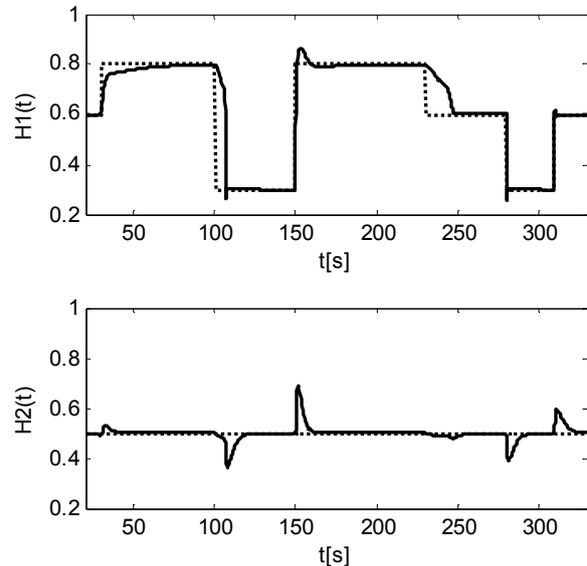


Fig.3. Step responses $H_1(t)$ and $H_2(t)$ in the system without decoupling for changes in set point value H_{SP1} . $Q_1=1.6474$, $Q_2=0.176$.

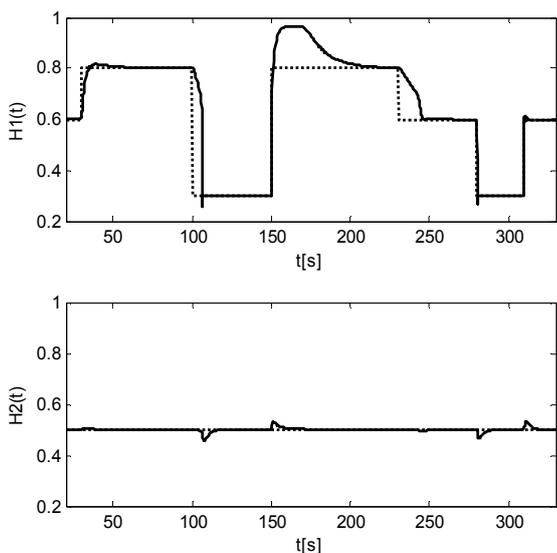


Fig.4. Step responses $H_1(t)$ and $H_2(t)$ in the system with partial decoupling of fast transients for changes in set point value H_{SP1} . $Q_1=2.22$, $Q_2=0.009$.

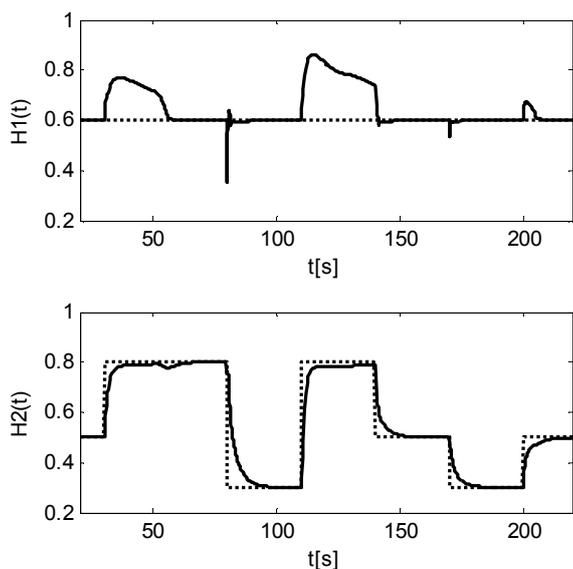


Fig.5. Step responses $H_1(t)$ and $H_2(t)$ in the system without decoupling for changes in set point value H_{SP2} . $Q_1=1.717$, $Q_2=0.647$.

The simulation results for changes of both set points have been shown in Figures (3-6), where the dotted lines denote set point values and the continuous lines denote process values. It is explicitly seen that in the system without decoupling there exist strong interactions between control loops (Fig. 3, 5) and they are stronger if changes of the level $H_2(t)$ for the constant cross section tank are larger (Fig.5) what is the result of the same relative degrees in the main and cross channels of the nonlinear plant. In spite of the fact that the decoupling matrix D_f was found for the linearized plant at the operating point ($h_{10}=0.6$, $h_{20}=0.5$), the use of the partial decoupling algorithm

gives significant reduction of the interactions between control loops for set point changes of both process variables $H_1(t)$ and $H_2(t)$ (Fig. 4, 6), even if the set point changes are quite large. However, for the system with decoupling mechanism one can notice some deterioration of the quality of the transients of $H_1(t)$ after the set point change H_{SP1} from 0.3 to 0.8 with reference to the same set point change H_{SP1} in the system without decoupling. The reasons of the deterioration are probably the improper DMC tunings, which were chosen for the nonlinear plant without decoupling mechanism. Hence, in the purpose of further improvement of the closed loop system one should choose the DMC tunings again taking into account the decoupling matrix D_f .

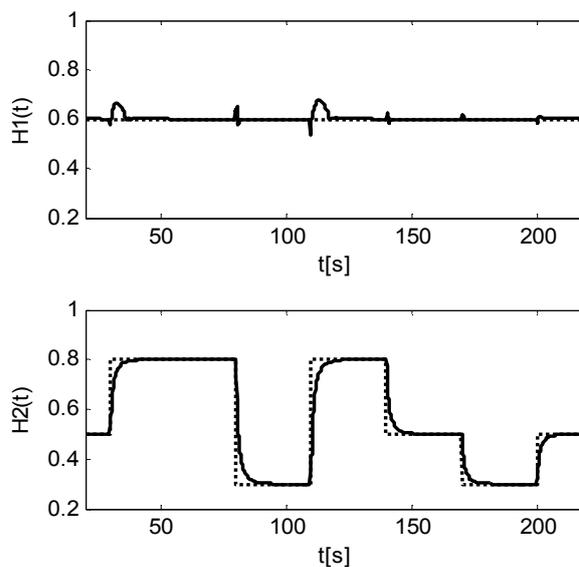


Fig.6. Step responses $H_1(t)$ and $H_2(t)$ in the system with partial decoupling of fast transients for changes in set point value H_{SP2} . $Q_1=0.0371$, $Q_2=0.46$.

VIII. DISTURBANCE REJECTION

In any real control system, there is always some amount of external disturbance; therefore each control scheme should take under consideration the influence of disturbances. Sinusoidal character of disturbance occurs commonly in process control; therefore this paper is focused on this type of disturbance. For the investigated control structure the detailed verification has been provided. The verification based on the large quantity of simulations that were necessary for drawing appropriate conclusions what constitutes important information for the designers of the control systems. The plant was disturbed by two additional oscillating sinusoidally with nonzero mean input flow streams: F_3 and F_4 , where the flow F_3 is the disturbance of the conical tank and the flow F_4 is the disturbance of the cylindrical tank. The influence of the disturbances has been studied for the both flow streams added separately for each tank and also for the both of them added simultaneously. When both disturbances were active the phase and frequency were varied and for the case of the single

disturbance only its frequency. Representative part of the results is presented below in Figures 7-10.

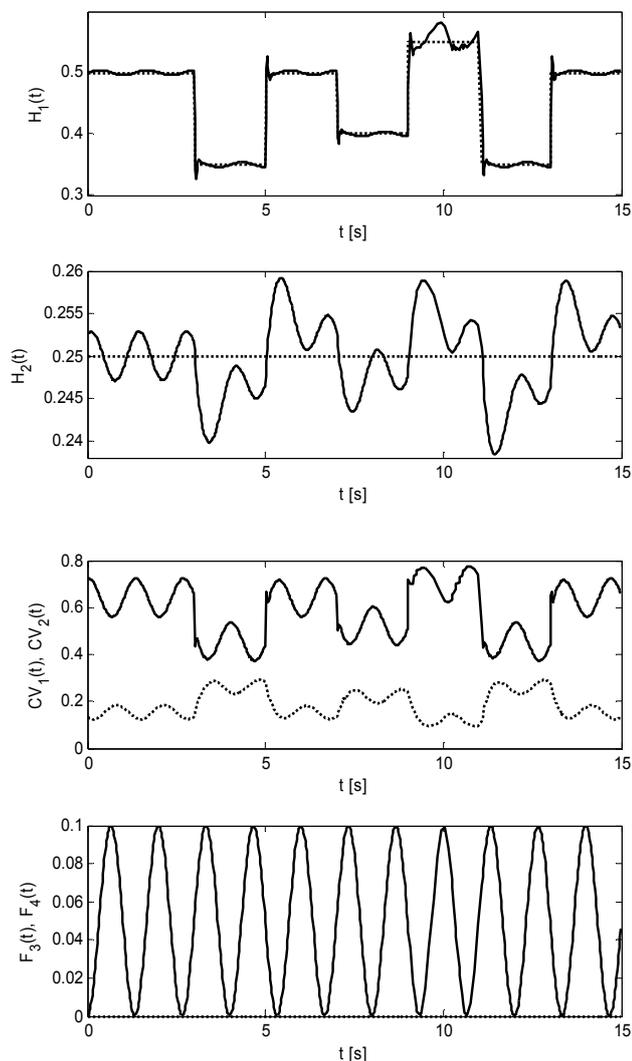


Fig.7. Step responses $H_1(t)$ and $H_2(t)$ in the system without decoupling for changes of set point value H_{SP1} and disturbance F_3 .

In Fig.7 and Fig.8 present the comparison between systems with and without decoupling for variable H_{SP1} and constant value $H_{SP2}=0.25$. This study presents situation where sinusoidal disturbance F_3 is added to the conical tank. In this case disturbance frequency is set to 0.75 Hz. However, during simulation experiments two different frequencies were taken under consideration and both of them were less than 1Hz. One of the frequency values is 0.25 Hz and the second one is equal to 0.75Hz like in the described case. In process control most disturbances which do not originate from sensors are slowly variable; therefore these two values were chosen. Magnitude (between peaks) of these sinusoidal disturbances is set to 10% (10% of whole range of process value changes). For presented situation for system with partial decoupling disturbance rejection of H_1 variable is visible better than in the system when decoupling mechanism is switched off. Figures 9 and 10

present situation where both disturbance flow streams are active. Additionally, the phases of flows are shifted. In spite of this situation, disturbance rejection for system with partial decoupling is still quite satisfactory especially for H_2 values.

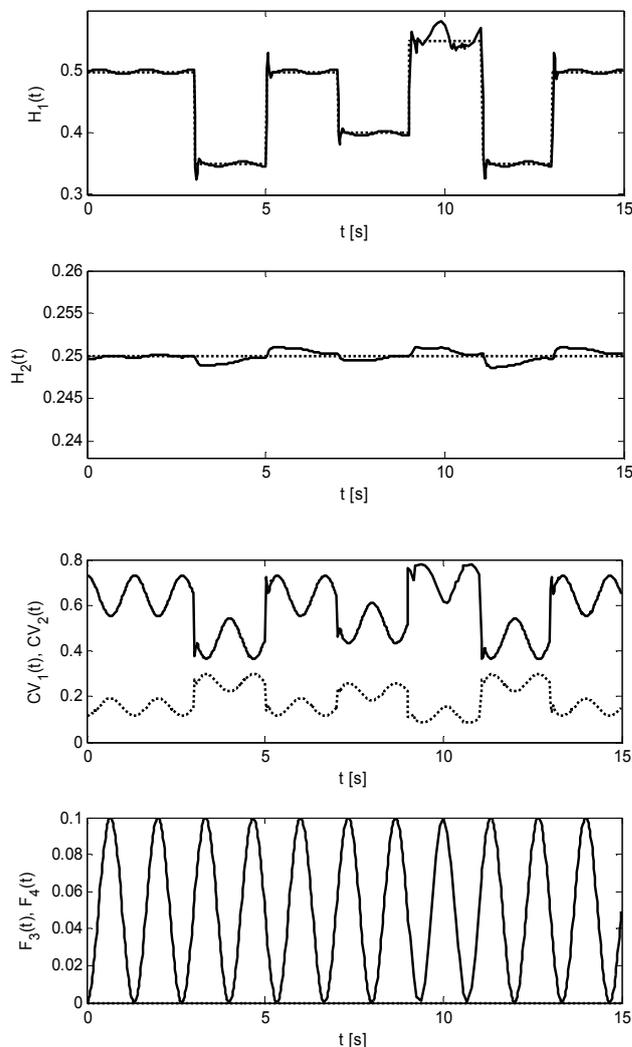


Fig.8. Step responses $H_1(t)$ and $H_2(t)$ in the system with decoupling for changes of set point value H_{SP1} and disturbance flow F_3 is turned on.

For better evaluation of the obtained results the paper shows only simulation results for disturbance frequency set to 0.75Hz. Results for disturbance frequency of 0.25Hz present similar properties as 0.75Hz. Therefore it was decided to omit 0.25Hz frequency in this work.

IX. CONCLUSION

Implementation of partial decoupling mechanism offers improvement of control quality in evident way therefore encourages one to future works on further research. The other advantages are that one does not have to know the accurate model of the plant and the decoupling matrix D_f does not

contain any dynamical elements. Thus, the implementation of the decoupling mechanism can be quite simple in the industrial control systems utilizing PLC devices. Generally, implementation of the decoupling mechanism provides better disturbance rejection, and for some range of the set points is necessary because control system has very weak disturbance rejection properties. The future works will focus on the practical validation of the proposed control strategy for the real laboratory installation. It is also important to test the partial decoupling algorithm for other nonlinear MIMO plants with stronger interactions between main channels to validate the usefulness of the algorithm.

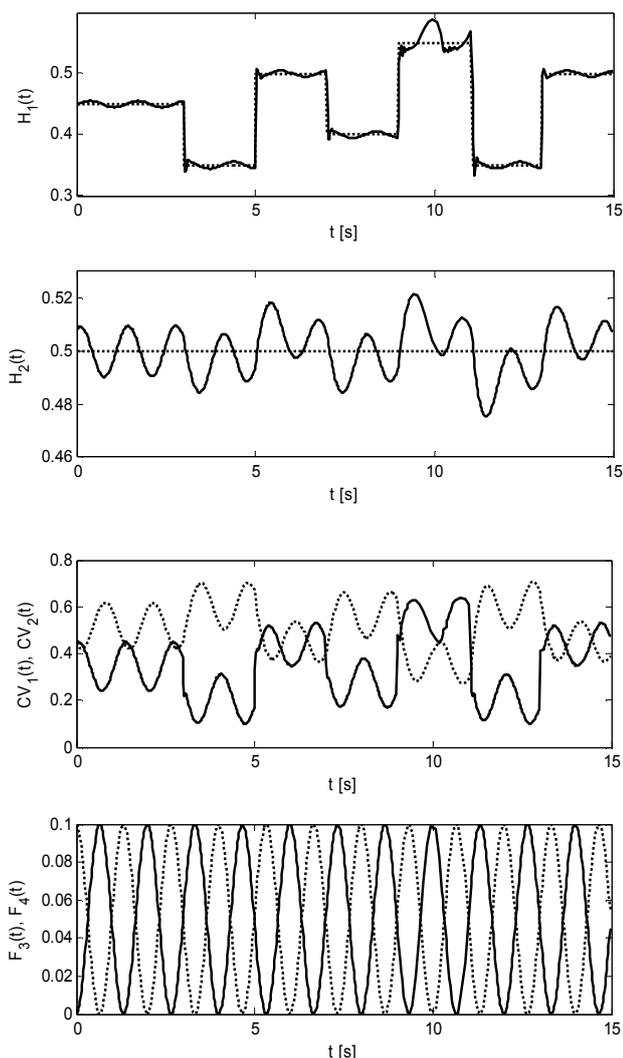


Fig.9. Step responses $H_1(t)$ and $H_2(t)$ in the system without decoupling for changes of set point value H_{SP1} and disturbance flow F_3 and F_4 is turned on.

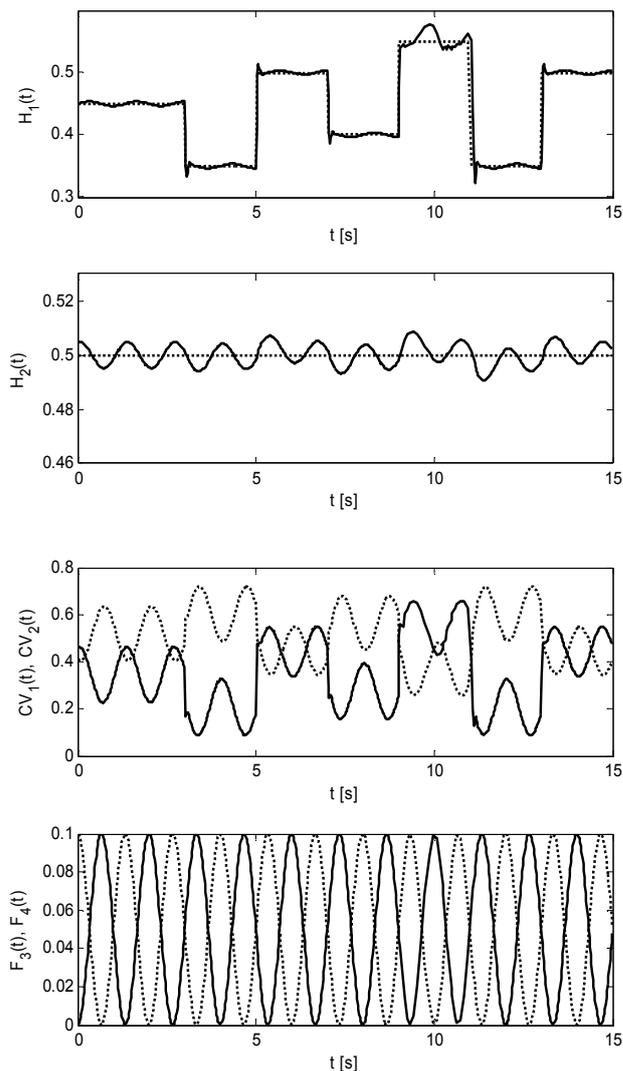


Fig.10. Step responses $H_1(t)$ and $H_2(t)$ in the system without decoupling for changes of set point value H_{SP1} and disturbance flows F_3 and F_4 are turned on.

REFERENCES

- [1] J. Czczot, P. Laszczyk, M. Metzger, "Local balance-based adaptive control in the heat distribution system - Practical validation", *Applied Thermal Engineering*, Vol.30, No.8-9, 2010, pp. 879-891.
- [2] C. R. Cutler, B. L. Ramaker, "Dynamic matrix control a computer control algorithm", in *Proceedings Joint Automatic Control Conference*, San Francisco, CA, 1980.
- [3] O. Camacho, L. Valverde, E. Iglesias, F. Rivas, "Dynamic Matrix controller based on Sliding Mode Control", In: *Proc. WSEAS MATH*, Harvard, Massachusetts, USA, March 24-26, 2008.
- [4] F. Garces, V. Becerra, C. Kambhampati, and K. Warwick, *Strategies for feed-back linearisation: a dynamic neural network approach*, London: Springer series Advances in Industrial Control, 2003.
- [5] K. H. Johansson. "The quadrupletank process: A multivariable process with an adjustable zero" in *IEEE Transactions on Control Systems Technology*, Vol.8, No.3, May 2000.
- [6] M. Metzger, *Hydraulic pilot-plant with strong nonlinearities*. Institute of Automatic Control - Activity Report (2005-2006), Editions of Silesian University of Technology, Gliwice, 2007, pp. 1-212.

- [7] E. F. Camacho and C. Bordons, *Model Predictive Control*, London: Springer, 1999.
- [8] P. Tatjewski, *Advanced Control of Industrial Processes, Structures and Algorithms*, London: Springer-Verlag, 2007
- [9] T. Kłopot, M. Metzger, „Simple Engineering Tuning of DMC Controller”, in *Proceedings of the 11th IEEE International Conference on Methods and Models in Automation and Robotics*, 2005 Miedzyzdroje.
- [10] T. Kłopot, P. Laszczyk, M. Metzger „Dynamic Matrix Control for Heating Process”, in *Proceedings of the 13th IEEE International Conference on Methods and Models in Automation and Robotics*, 2007, Miedzyzdroje
- [11] D. Dougherty, D. J. Cooper, “A Practical Multiple Model Adaptive Strategy for Single-Loop MPC”, *Control Engineering Practice*, Vol.11, 2003, pp. 141.
- [12] N. Bhuvanewari , G. Uma, and T. Rangaswamy, “Adaptive and optimal control of a non-linear process using intelligent controllers”, *Applied Soft Computing*, Vol.9, No.1, 2009, pp. 182-190.
- [13] K. J. Aström, B. Wittenmark, *Adaptive Control*, Addison Wesley USA, 1995
- [14] G.C. Goodwin, S.F. Graebe, M.E. Salgado, *Control system design*, Prentice Hall Upper Saddle River, NJ, 2001
- [15] S. Hadisupadmo, R.J. Widodo, T. H. Soerawidjaja, H. A Tjokronegoro, “Identification Process, Design and Implementation Decoupling Controller for Binary Distillation Column Control”, in *WSEAS Transactions on Electronics*, p353-358, Issue 2, Vol. 1, April 2004
- [16] P. L. Falb, W. A. Wolovich, “Decoupling in the design and synthesis of multivariable control systems”, in *IEEE Transactions on Automatic Control*, Vol.12, 1967, pp. 651–659.
- [17] W. M. Wohnam, S. Morse, “Decoupling and pole assignment in linear multivariable systems: a geometric approach”, in *SIAM Journal on Control and Optimization*, Vol. 8, 1970, pp. 1–18.
- [18] D. Claude, “Decoupling of nonlinear systems”, *Systems & Control Letters*, Vol.1, 1982, pp. 301–308.
- [19] I. J. Ha, E. G. Gilbert, “A complete characterization of decoupling control laws for a general class of nonlinear control systems”, in *IEEE Trans. Automat. Contr.*, Vol. AC-31, 1986, pp. 823–830.
- [20] A. Isidori and J.W. Grizzle, “Fixed modes and nonlinear noninteracting control with stability”, in *IEEE Trans. on Autom. Contr.*, Vol.8, 1988, pp. 345–349.
- [21] J.C. Quadrado, “Decoupling of an EV active steering and direct yaw moment control systems”, in *WSEAS Transactions*, 2004.
- [22] L. Ciprian , P. Dumitru , U. Andreea , D. Catalin, “Solutions for nonlinear multivariable processes control”, in *WSEAS Transactions on Systems and Control*, Vol.3 No.6, p.597-606, June 2008.
- [23] R. Gessing, “Implementability of Regulation and Partial Decoupling of MIMO Plants”, presented at 14th Mediterranean Conference on Control and Automation, 2006.
- [24] P. Skupin, “Partial decoupling of steady states and fast transients in multivariable systems”, M.S. thesis, Dept. Automatic Control, Electronics and Computer Science, Silesian University of Technology, Gliwice, Poland, 2008.
- [25] M. Metzger, „Comparison of the RK4M4 RK4LIN and RK4M1 methods for systems with time-delays”, *Simulation*, Vol.52, No.5, 1989, pp. 189-193.
- [26] M. Metzger, “A comparative evaluation of DRE integration algorithms for real-time simulation of biologically activated sludge process”, *Simulation Practice and Theory*, Vol. 7, 2000, pp. 629-643.