Applications of the Four Color Problem

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Abstract—In this paper are followed the necessary steps for the realisation of the map’s coloring, matter that stood in the attention of many mathematicians for a long time. It is debated the matter of the four colors, but also the way of solving by implementing of an algorithm in the MAP-MAN application. Also, it is tackled the maps drawing in real time within GPS system satellites, using more colors depending on the covered route and the landforms met.

Keywords—The issue of the fourth colors, The MAP-MAN application, Software for GPS.

I. INTRODUCTION

The issue of the map’s coloring with four colors it dates since 1852 when Francis Guthrie, while he intended to color the map of the England regions, he ascertain that four colors are enough. Four color problem has all valences of a great career problem: first its formulation is extremely simple, requires no mathematical knowledge; secondly, it has remained unresolved for over a century, is a surprisingly bad and raised concern of many famous mathematicians. Let us look at some of the partial results of the demonstrate theorem. The fact that three colors are not sufficient for coloring any map plan was quickly found (see Fig.1a). De Morgan has shown that there is no map formed of 5 regions so that they are neighbours two by two, so it can be colored with four colors. AB Kempe, a member of the London Mathematical Society and extremely passionate about mathematics has published in 1879 an article which claimed that he has proved the theorem. His reasoning was very clever; he reduced the problem to normal maps, i.e. maps that has no country completely closed in other countries and no points where meet than three regions. Although the reasoning was incomplete, it contains the basic ideas that led to a correct demonstration a century later. Thus, you can associate a graph to any map in which each region is represented by a point and two points will be connected by an edge if and only if the points correspond to two neighbouring regions (see Fig. 1b). [1-6], [9-14]. In this way the coloring areas on the map problem is the problem of coloring the graph associated points so that points connected by an edge are colored differently. Four color problem has contributed to important research in graph theory, such as chromatic numbers of graphs.

With a particular graph, mathematician PJ Heawood showed in 1890 that Kempe demonstration has an error not easy to remove. Later in 1913, the mathematician Ph. Franklin of the Massachusetts Institute of Technology raised the number of regions for which the problem is solved from 5 to 21, and in 1940 Winn manage to reach 35 regions. Another particularly interesting result is due to PJ Heawood, who has devoted 60 years of his life studying the problem. Here it is: the probability of finding a map with more than 36 regions that can not be colored with four colors is less than 10^-1000! (note that 10^1000 is a number greater than the number of atoms in the entire galaxy ...). A theorem of the five colors (five colors that are effective to color a map) was obtained relatively easily, an elementary demonstration of this result can be found in "About numbers and figures" of H. Rademacher and O. Toeplitz. If it were possible to produce all these configurations and show that they can be colored with four colors, then the demonstration was complete. The most efficient method of configuration production proved to be a computer algorithm implemented by W.Haken and K.Appel, University of Illinois, who spent almost 1200 hours and, finally, the demonstration was completed [16]. A year later, using a different procedure to reduce unavoidable configurations, F.Allaire of the University of Waterloo, Ontario, CA, was able to demonstrate the theorem in just 50 hours of human-computer dialogue. Naturally aroused enthusiasm in the world of mathematicians of this unusual success until then was tempered by sceptical voices that said that this is not a mathematical theorem in the classical sense. Thus, T.Tymoczko in the article “Four color problem and its significance philosophic” (Journal of Philosophy, 1979) theorem says that express a retrospective and not really a priori, like any mathematical truth. His arguments are based primarily on failure to check manually (with pen) the demonstration, given that there is a unique algorithm to determine all possible computer programs. In response, ER Swart writes in [17] reported that the inconvenience is apparent, because very many calculations performed on the computer were routine and the program used can be verified. Computer task was overwhelming in size, charge that the man knew how to deal with, but it could never end. That was the first memorable event from the world of mathematicians that had to admit the existence of partially accessible human
demonstrations and computer law to help us in establishing mathematical truths.

II. THE MATTER OF THE MAP COLOURING WITH FOR COLOURS

The problem definition. Beeing given a map (more neighbouring areas among them), on demand to color the map with outside for different colors, therefore any from that two neighbouring areas to be colored different [15].

The problem framing. The problem makes part from the category of the constraint problems of (“constraint – satisfaction problem”, a.k.a. CSP)

Useful observation. The map can be seen like a graf in which:
- every knot suits to an area on the map;
- every edge suits to a bond from two countries.

III. THE SOLUTION OF THE COLORING

The solution algorithm of the coloring problem of a map (intelligible part of the application) was realised using the CLIPS language [7].

The graphic interface of the application was developed using the orientation – C# object [8].

The coloring program of a map, realised in the CLIPS declarative language, is formed from two parts:
- the so called algorithm of coloring wich is made from rules (rules) – to be seen the „mapping_engine.clp”,
- the data structures used by the algorithm which are represented by fact (facts) – to be seen the „mapping_facts.clp”

The implementation in CLIPS is using three specific makers of:
- deftemplate – for the definition of the data structures (the definition of the fact template),
- deffacts – for the data definition (facts),
- defrule – for the rules definiton (the rules are executed actions over the facts if are observed some condition).

A rule in CLIPS it is made from two sides (sides):
- RHS (Right-Hand Side) – specific for the so called action who is usually executed,
- LHS (Left-Hand Side) – specific for the condition who must be fulfil (the moment) for executing the action RHS.

The MAP-MAN application is different from the classic case by the fact that it mustn’t colored all the countries on the map.

The graphic interface allows to the user to select the countries that will be colored by the algorithm. Follow-up, we will use the next terms:
- the active country – nominates a country with the user select (by clicking on it) for being colored;
- the inactive country – nominates a country which mustn’t be colored (the user didn’t clicked on it);
- the validate color – nominates a color associated to a country which has a number smaller than 4 and who is different from the colors of the neighbouring countries (a color is a whole value between 0 and 3).

A. The data structures (Templates, Facts). Templates:
<country>: it is use for specify the fact’s format (the structure) of the <country>, are the following fields:
<code> - the whole positive value, who specify the „name” of the country;
<color> - the whole value (-1) and 3, who nominates the „color” associated to a country;
- an uncolored country has the color field is = -1 (initially, all the countries has the color is = -1);
<flag> - the whole positive value, between (-1) and the number of the three active countries;
- for an inactive country we will have: flag = -1;
- used in the backing phase of the algorithm;
<was_first> - can take two values: on/off;
- used in the moment when the algorithm gave back until the first country that he colored, for that the algorithm not choose again a country which was already choose the first in the begging (to avoid an infinit curl);
- initially, every country has was_first = -1;
- if a country was chosen the first (meaning that there aren’t other active colored countries on the map), then she will have was_first=on (a complete name of this field would be <was_first_colored>);

< neighborhood> - a multislot field, used for specify the neighbors of a country.

Facts:
- <no of active countries> - specify the number of active countries;
- <flag_counter> - used in the backing phase of the algorithm;
- <next_country> - the country for which we try to associate a validate country
- <next_color> - nominates the candidate’s color for the country who has <code> equal with „next_country”.

B. The algorithm (Rules). The algorithm uses the next nine rules:

Rule 1: <set_first_next_country>
LHS (the condition / the ignition moment): doesn’t exist other active colored country;
RHS (actions): - choose a country, from those with the field was_first=off, to be the first colored;
- set was_first = on for the chosen country;
- put on 0 the candidate color (next_color=0) for this country.

Rule 2: <main_final>
LHS: after all the active countries were colored;
RHS: get the ignition of the display rule fo results.

Rule 3: <print_map_colors>
LHS: after the RHS execution from Rule 2;
RHS: result’s displaying.

Rule 4: <main_loop_when_not_backing>
LHS: after select another country that must be colored;
RHS: for the country with the field <code> equal with „next_country” put the field <color> on 0.

Rule 5: <main_loop_when_backing>
LHS: when we didn’t find the country that it must be colored („next_country”) a validate color (<4); this rule is not active if it isn’t colored at least one country (in this case interfere the Rule 1);
RHS: add the associate color to the last olored country.

Rule 6: <color_is_valid>
LHS: when the candidate color („next_color”) for the country that it must be colored („next_country”) it is validate (meaning between 0 and 3 sand different from the color of any neighbour);
RHS: actualise the fields <color> and <flag> for the country „next_country”; also, actualise the fact <flag_counter> that it will be use in the backing phase of the algorithm (meaning that then when the algorithm recoil).

Rule 7: <invalid_color>
LHS: when for the country that it must be colored („next_country”) the candidate color („next_color”) it is not validate;
RHS: get the ignition of the Rule 8 (rather, creates a fact <inc_color> for this).

Rule 8: <increment_color>
LHS: after the RHS execution of the 7 rule (meaning after the creation of an abject <inc_color>);
RHS: add the candidate color („next_color”) for the country that it must be colored at the current frequentative (meaning for „next_country”).

Rule 9: <color_is_4>
LHS: when for the country that it must be colored („next_country”) the candidate color („next_color”) was added until it reach the 4 value (validate value; the validate values are 0, 1, 2, 3);
RHS: - actualise the fields <color> and <flag_counter> of the country „next_country” (color = -1; „flag=flag + 1”),
- actualise the field <flag_counter> („flag_counter=flag_counter -1”),
- create a fact (<back>) for the ignition of the 5 rule (<main_loop_when_backing>).

C. The graphic interface (C#). The graphic interface of the application allows the user to open a folder with the extension „.map”.
The folders with this extension are, actually, maps that respects a certain format (this format will be explained later).
The pening of a amp using the menu is made in this way: File->Open map.

After the map was opened, the user will click (pushing on the left button of the mouse) on the countries that he wants to color by the algorithm.

To start to color the map, we will use the menu: Compute->Start.

Every country that the user clicked will initialy be colored orange.

After the map’s coloring command, the countries on which we clicked will be colored with red, yellow, green and blue [15].

The format description <.map>.
The MAP folders are text folders, having the next:
- Line 1: contains only ohne number that indicates
the polygon number (countries) present on the map
- Line 2: contains 2 numbers representing the co ordinates X and Y of the map (these numbers are used at the map’s scalaring);
- The next lines: on every line is discribed an
polygon under the next form:
- the first column contains “his name” (meaning that a whole number; e.g.: 1, 2, 3, 4, … 7);
- the second column contains the point number from which it is formed the polygon (it doesn’t exist a relative limitation at the point’s number of the polygon);
- then it follows 2 columns representing the co-ordinates X and Y of the polygon’s points;
- the next column states the neighbouring’s number of the polygon describend by the respective line;
- the last columns from one line states the neighbours of the polygon describend by the respective line.

Observations:
- Pay attention at the spaces and at the tabs: between any 2 elements of one line it is a space or a tab. If we don’t respect this rule it is highly that the program not working correctly;
- The map it is made from polygons of any dimension. It is rather that this polygons to not be concave, because it can be problems at the polygon’s detection where we clicked.

The folders of the .map’s type allows the construction of a map very different.

The MAP-MAN program reads such kind of folders and on the basis of them constructs the graphic maps (we can say that the .map’s folder contains maps in text format, which MAP-MAN converts them in graphic format).

IV. CODE LISTING

The Code listing supposes the following stages:
A. The template definition:
   <country> (the <mapping_engine.clp> folder).
B. The facts definition that describes the map (this side of the CLIPS program differs from a map to another; further, it is presented an exemple).
C. The rules definition (the so called algorithm).

Code listing:

; Rule 1 (defrule set_first_next_country;
  ; (no_of_active_countries ?noac) (flag_counter ?fc) (next_color ?nc)
  ; (assert (flag_color +1?)) (modify ?fc (+1?fc)) (retract ?inc) (assert (flag_color +1?))

; Rule 2 (defrule main_final
  ; (no_of_active_countries ?noac)
  (flag_counter ?fc) (flag_color ?c) (modify ?fc (+1?fc))

; Rule 3 (defrule print_map_colors
  ; (print on) (country (code ?code) (color ?c) ) => (printout t "Country" ?code "has the color" ?c crlf))

; Rule 4 (defrule main_loop_when_not_backing
  ; (not (exists (back ?)) ; (flag_counter ?fc) (test (<0?fc))
  ; We exclude the case when we don’t have any colored active country
  ?ic<- (country (code ?next_country) (color -1) ?flag)
  (test (<= 0 ?flag)); if(?flag>0,
  Meaning that the country is "active"=>
  (assert (next_country ?next_country))
  (assert (next_color 0)) (modify ?c (color 0)))

; Rule 5 (defrule main_loop_when_backing
  ; (flag_counter ?fc) (test (<0?fc))
  ; We exclude the case when we don’t have any colored active country
  ?ic<- (country (code ?next_country) (color ?c) ?flag)
  (test (and(<= 0 ?flag) (=?flag (+1?flag_counter)))
  if(0<=?flag = ?flag_counter - 1) =>
  (assert (next_country ?next_country))
  (assert (next_color (+1?c))) (modify ?c (color (+1?c)))
  (retract ?ib))

; Rule 6 (defrule color_is_valid
  ; (declare (salience 7));
  ; (forall (country (code ?next_country) (color ?candidate_color))
  ; (test (< ?candidate_color 4))
  ; (forall (country (code ?next_country) (neighborhood$ ??nbour $))
  ; (exists (country (code ?nbour) (color ?candidate_color))))
  ; (retract (inc_color off)) (modify (inc_color on)) (assert (inc_color on))

; Rule 7 (defrule invalid_color;
  ; (exists (country (code ?next_country) (color ?candidate_color))
  ; (test (=?flag_counter (+1?flag_counter)))
  ; (assert (next_country ?next_country))
  ; (retract (inc_color off))

; Rule 8 (defrule increment_color;
  ; Is adding the color (of a country) if it is <4
  ; (forall (country (code ?next_country) (color ?candidate_color))
  ; (test (=?flag_counter (+1?flag_counter)))
  ; (assert (next_country ?next_country))
  ; (retract (inc_color off))
  ; (modify (inc_color on))

; Rule 9 (defrule debug
  ; (assert (print on)) (country (code ?code) (color ?c) ) =>
  ; (printout t "Country" ?code "has the color" ?c crlf))

; Rule 10 (defrule debug_next
  ; (assert (next_country ?next_country))
  ; (assert (next_color (color 0)))

; Rule 11 (defrule debug_prev
  ; (assert (prev_country ?prev_country))
  ; (assert (prev_color (color 0)))

; Rule 12 (defrule debug_color
  ; (assert (color ?c)) =>
  ; (printout t "Color" ?c crlf))
Predicate logic:
(test (?candidate_color 4)) => (retract ?i_inc)
(retract ?i_next_color)
(assert (next_color (+ ?next_color 1)))
(modify ?ic (color (+ ?candidate_color 1))))

Rule 9 (defrule color_is_4;
Lights up when we didn’t find an validate color for a country (<4)
(declare (salience 7));
?in <- (next_country ?next_country)
?f <- (flag_counter ?flag_counter)
?ic <- (country (code ?next_country) (color 4) ); (flag 0))
?inc <- (next_color ?next_color) =>; backing...
(retract ?in)  ; 1) delete next_country
(modify ?ic (color -1) (flag (+ 1 ?flag_counter)));
2) restore country
(retract ?f); 3) decrement flag_counter
(assert (flag_counter (- ?flag_counter 1))
(retract ?inc); 4) (assert (back on)))

V. SNAPSHOTs

Further, on presents the results which we obtain by the execution of the coloring algorithm for the map described by the harta_wilensky.map folder
A. Map-Man after the folder opening.map (Fig. 3).

B. Map-Man after the user clicked on some countries (Fig. 4).
C. Map-Man after the algorithm execution of the map coloring (Fig. 5).
D. The folder’s listing harta_wilensky.map:

```
7
680 420
1 4 0 420 0 351 680 420 6 2 3 4 5 6 7
7 5 0 350 0 400 0 400 90 90 350 4 1 2 3 6
```

VI. ROMANIA AND GREECE MAP

Speaking of the map, JayIsGames, blog where I first read about this game, says that to color the U.S. map so that two adjacent states are not the same color you need ... four colors. I wonder how many Romanian counties map color needs?

Formulating theme. The objective of this section is to implement a program addressing the problem of satisfying constraints. The chosen theme is to color a map of Romania and Greece with four colors so that the side by
side surfaces to be colored differently.

**How to solve.** Solving the problem consists in using two programs:
- The "smart" is implemented in Clips;
- Visual interface is implemented in a high-level procedural language. Visual C#.

Communication between these programs is achieved through two dll files: Clips.dll and ClipsWrapper.dll

**Detailed description.** The program has in its three modules made in clips (main.clp, search.clp and map.clp) modules that perform the actual problem and find the solution.

- main.clp - contains three deftemplates:
  1) pb-describe the current status of the problem - has three slots: - state (describe the current status of the problem and can have six values: start, in-progress, new-depth, choice-var, solved, blocked), level (mean the level of recursivity) and hypothetical-var (hypothetical variable is chosen to the current search); 2) var-describes each variable of the problem, has four slots: name (the name of the variable), level (clamp level), possible-values (the list of possible values attached to a variable) and value (variable value ->color of the city); 3) solution - used to the counting of solutions and their saving in variable-value (var-val) - contains 5 rules: all-solutions? (start search solution), analysis (search in progress), save-solution (saving solution in the form - var-val), print-solution (display solutions) - search.clp - contains six rules:
  1) generate-new-depth (generates a new level search);
  2) choice-var-val (choose a hypothetical value for a new level of search). It is chosen the variable with the fewest possible remaining values and is chosen as a possible value the first in the list);
  3) instantiation (produce an analysis if there are any unused variables and the list of possible values only have one value, it is assigned);
  4) impasse – is blocking the algorithm when the solution that we follow is a wrong one;
  5) backtrack-on-value (choose using backtracking algorithm a new value for the current variable);
  6) backtrack-on-variable (choose using backtracking algorithm, a new value from the previous step for the current variable);
- map.clp - contains the actual data of the problem (possible colors for each city, each city neighborhood and the test rule that two neighboring cities do not have the same color).

Results obtained by running the program in different situations.

Table 1: Run with 4 possible values for coloring.

<table>
<thead>
<tr>
<th>City</th>
<th>1st solution color</th>
<th>2nd solution color</th>
<th>3rd solution color</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Blue</td>
</tr>
<tr>
<td>1</td>
<td>Blue</td>
<td>Green</td>
<td>Yellow</td>
</tr>
<tr>
<td>2</td>
<td>Magenta</td>
<td>Magenta</td>
<td>Magenta</td>
</tr>
<tr>
<td>3</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
</tr>
<tr>
<td>4</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
</tr>
</tbody>
</table>

Listing important parts of the source code:

**search.clp**

```plaintext
(defrule SEARCH::backtrack-on-value
  :when (state "blocked" for the chosen hypothetical-value ?x we must choose another value from the possible values list)
  :then (solve ?x)
)
```

(focus PROPAG)

```
(defrule SEARCH::backtrack-on-variable
  :if problem’s solve is blocked and there are no more possible-values for the hypothetical-variable, we must return to the previous search level where solving wasn’t blocked and we choose another value from the possible-values of this level.
  :then (solve ?x)
)
```

(map.clp)

```
(defrule PROPAG::different colors for neighbours
  :if ?x has associated the color ?v then we eliminate ?v from the possible-values list of ?y nodes neighboring with ?x
  :then (node(name ?x)(value ?v)(level ?m))
)
```

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identify ?x neighbours
\( \Rightarrow (\text{bind}\, \text{poz_cul}(\text{member}\, ?y\, \text{?liste})) \)
(if \( ?m \neq ?n \) then (modify?if(possible-values
(delete$?lists?poz_cul?poz_cul)))
else (duplicate?if(level?n)(possible-values
(delete$?lists?poz_cul?poz_cul)))))

![Map of Romania](image)

**Fig. 6:** The coloring of the Romania map: 4 culors and 5 coulors.

**Table 2:** Run with 5 possible values for coloring.

<table>
<thead>
<tr>
<th>City</th>
<th>1st solution color</th>
<th>2nd solution color</th>
<th>3rd solution color</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Blue</td>
</tr>
<tr>
<td>7</td>
<td>Yellow</td>
<td>Yellow</td>
<td>Yellow</td>
</tr>
<tr>
<td>8</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
</tr>
<tr>
<td>9</td>
<td>Cyan</td>
<td>Cyan</td>
<td>Cyan</td>
</tr>
<tr>
<td>10</td>
<td>Magenta</td>
<td>Magenta</td>
<td>Magenta</td>
</tr>
<tr>
<td>11</td>
<td>Blue</td>
<td>Blue</td>
<td>Blue</td>
</tr>
</tbody>
</table>

| 1    | Blue               | Green              | Yellow             |
| 2    | Magenta            | Magenta            | Magenta            |
| 3    | Cyan               | Cyan               | Cyan               |
| 4    | Cyan               | Cyan               | Cyan               |
| 5    | Blue               | Blue               | Blue               |
| 6    | Blue               | Blue               | Blue               |
A. List of main classes for implementing map of real-time (GPS)

1) CTransfert Class - this class, common server and client, manages the transfer information (messages, actions, tables of measures) between the server and the patient. His role will be to serialize the information in the

Fig. 7: The coloring of the Greece map: 4 culors and 5 coulors.
form of a compressed frame that the client will decode.

The main methods of this class Class2Byte and Byte2Class. The first serialized all of the information in the fields of the class (action, size and content of tables of measures, display format, News ...) in a first stream, then this will compress flows.

The Byte2Class method is the reverse of the previous one, which will decompress the flow specified before completing all fields in the class with the values in the frame. The table’s measures, if any, will also be established so that the main class can access thereafter.

B. Server. Device Class. It realizes the interface between the main class and class transfer, managing the applications and by providing the class transfer relevant information to form the frame.

The main method of class, TraiteDemande responsible action to address sent by the client. It returns a feed store and its length, corresponding to the answer frame to be sent to the patient.

If an acquisition is made of measures, the method fills an array of complex Class CTransfert.

There are also actions settings current settings of the device, contained in the fields which will be sent to the device to match demand in real setting. Indeed, for certain input parameters, the user manually enters a value.

According to the button pressed, our class will simulate the actual behavior of the device and must accurately update the contents of current setting.

Once the treated TraiteDemande launches flow generation, conducted in the classroom CTransfert, then returns the flow compressed the main class.

TmainForm Class - class that declares the main menu of the application server, menu items and manages the events that occur in the form.

Be careful methods AppSocketPatient-Connect, AdminSocket PatientConnect, AppSocketPatient-Read, Admin SocketPatientRead which are methods of learning peeled event when receiving a connection from a client or receiving a string representing a client’s message.

It was decided to separate information about the connections, logins ... (Admin) messages of each device (App) into two sockets to facilitate management.

At each new connection, the server verifies that it is a valid user with the password he compares to the list included in the file "serveur.ini." If it finds a match and if this login is not already connected, it sends a message to the opening of the connection [20].

Otherwise, it returns a message of non-authorization and closes the socket connection.

Each action received for the patient order, then we run the method of TraiteDemande of the Camera class: this action is decoded and the response to the motion is generated in the form of data stream.

Once the action is treated on referral response sockets patient transmitter and receiver doctor.

Another important method is the ChangeEtatPatient called in a double click on the login of a patient, and that changes the status of user choice and, if necessary, other users (because he can not have more than one user issuer at a time).

C. Patient. TMainForm Class - class that declares the main menu application main patient, menu items and manages the events that occur in the form.

He asked his brother Frederick if it is true that any map could be colored using just four colors, therefore the abutting regions (meaning those who has a segment of common edge, not just one point) correspond different colors. Frederick Guthrie communicate to DeMorgan this hypothesis. The first allusion at this issue was printed due to Cayley (1878). A year later appered the first “demonstration” made by Kempe. The errors of this demonstration were punctated 11 years later by Heawood. Another demonstration who failed it is due to Tait (1880); an empty space left in this argumentation was dot by Petersen in 1891 [4], [6], [13]. Those failed demonstration didn’t have any value even that Kempe discovered that was becoming known under the name of "the Kempe’s handcuffs", and Tait discovered an equivalent wording of the Four Colors Theorem in the coloring terms of three edges [1], [2]. The next major contribution is brought by Birkhoff who’s job allowed to Franklin in 1922 to prove that the hypothesis of the four colors is true for maps with at least 25 regions [5]. Also, she was used by other mathematicians for improving For Colors Issue. It should mentioned aside Heesch who developed the two principal ingredients necessary to the last demonstration reducibility and discharge. Also, until the concept of the reducibility was studied by other researchers, it seems that the discharge idea, crucial for an unavoidable part of the demonstration, it is due to Heesch, he was the one who launched the hypothesis that a proper development of this method would solve the Four Colors Theorem. The Heesch hypothesis was confirmed by Appel and de Haken in 1976, in that year they published their own demonstration of the Four Colors Theorem [3], [9], [10], [11], [12], [14].

VII. CONCLUSIONS

MAP-MAN is an application wich propose the implementation of the coloring issue of a map, using a graphic interface. The application solves the two main problems - the reducibility and the discharge. If the four colors problem is a static one and it was solved through the application MAP-MAN, the problem of maps drawing in real time within GPS system satellites is a dynamic, real time and current one, and can be easily solved by using oriented object programming as it has been shown in 6.1 paragraph.

REFERENCES

128, 1913.


